Chapter 10
Probability

10.1 Sample Spaces and Probability

Probability Experiment

- A probability experiment is an action, or trial, that has varying results.

GOOD

Outcomes

BAD

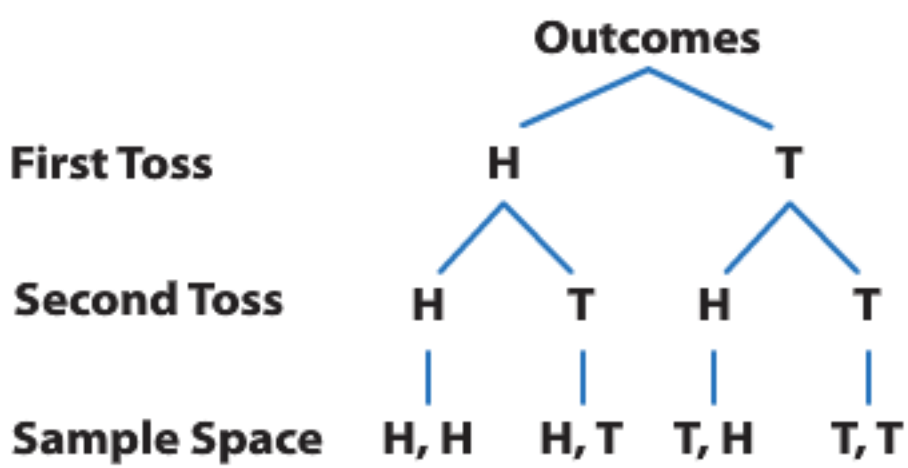
10.1 Sample Spaces and Probability

Vocabulary

- **Outcome** - The possible results of a probability experiment.
- **Event** - A collection of one or more outcomes.
- **Sample Space** - The set of all possible outcomes.

Example: Flip a coin twice.

Tree Diagram →



10.1 Sample Spaces and Probability

Experiment: Roll two 6-sided dice

- **Sample Space** - The set of all possible outcomes: roll 1/1, or 1/2, or 1/3, …, or 6/4, or 6/5, or 6/6

- **Event** - A set of outcomes, usually expressed as a capital letter (e.g. A = "sum of dice = 7")

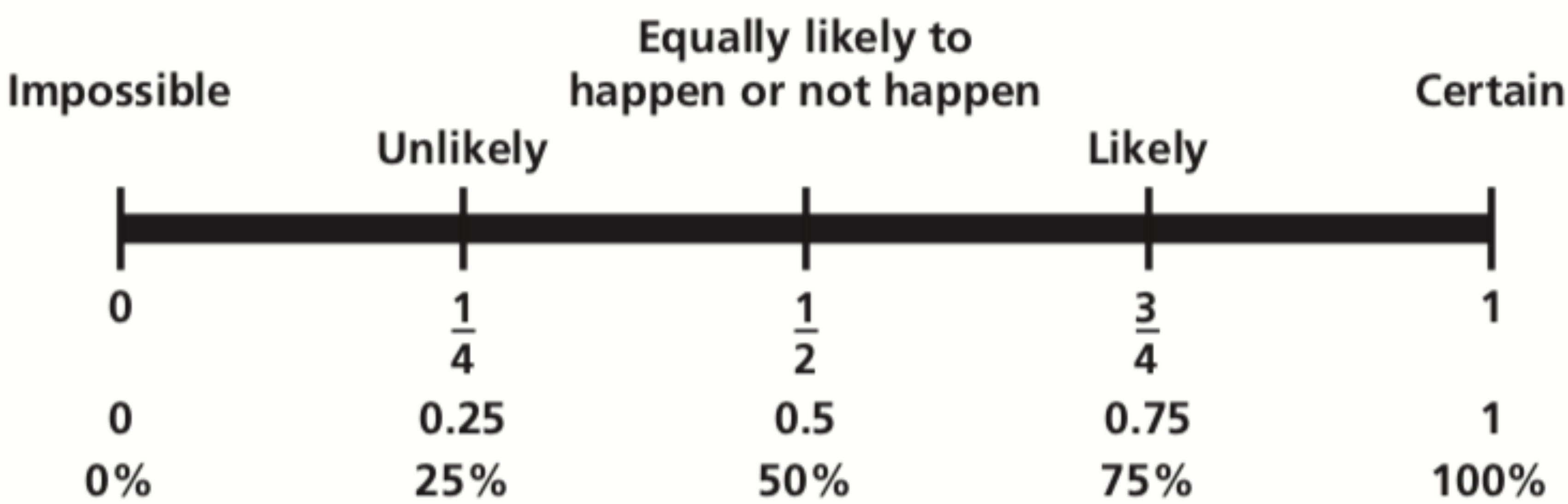
- **Probability of an Event** - The chance that an event "A" will happen or P(A).

Possible Sums		First Die					
		⚀	⚁	⚂	⚃	⚄	⚅
Second Die	⚀	2	3	4	5	6	7
	⚁	3	4	5	6	7	8
	⚂	4	5	6	7	8	9
	⚃	5	6	7	8	9	10
	⚄	6	7	8	9	10	11
	⚅	7	8	9	10	11	12

10.1 Sample Spaces and Probability

Probability of an Event

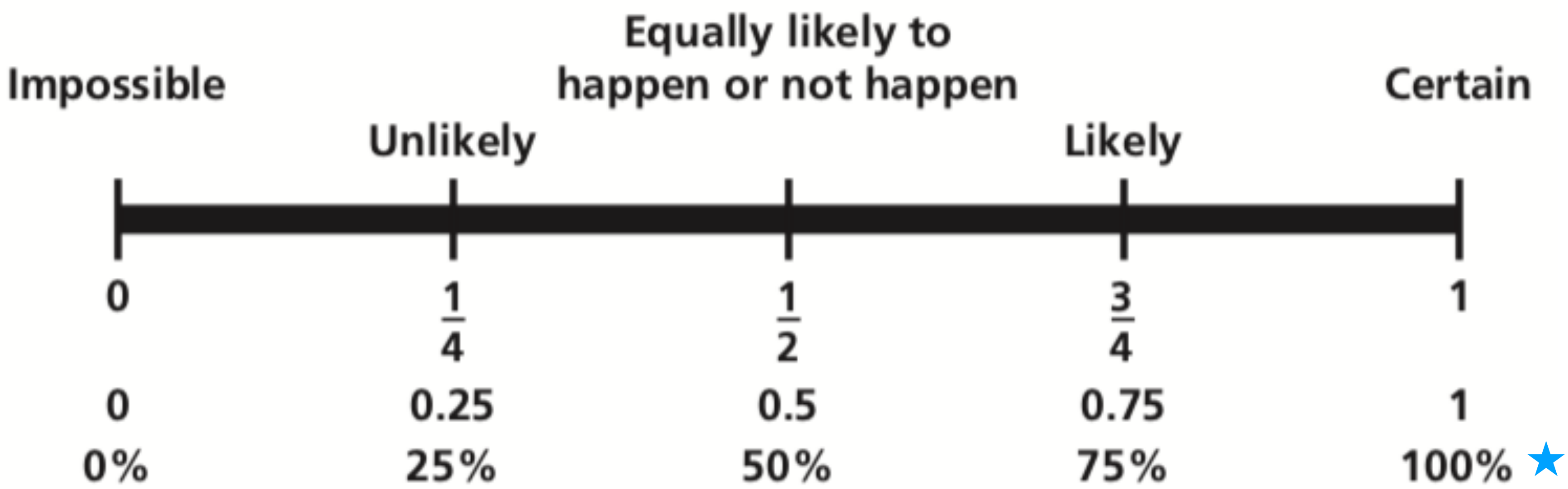
- A measure of the likelihood, or chance, that the event will occur.
- Probability is a number from 0 to 1, including 0 and 1, and can be expressed as a decimal, fraction, or percent.



10.1 Sample Spaces and Probability

Calculating Probability

$$\text{Theoretical Probability} = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$$



10.1 Sample Spaces and Probability

Experiment: A student guesses on four true/false questions. What is the probability the student will make exactly two correct guesses?

- **Build a Model** - The table below represents incorrect (I) and correct (C) answers.

Number correct	Outcome
0	IIII
1	CIII ICII IICI IIIC
2	IICC ICIC ICCI CIIC CICI CCII
3	ICCC CICC CCIC CCCI
4	CCCC

exactly two correct → 2

$$\frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}} = \frac{6}{16} = \frac{3}{8} = 37.5\%$$

The Complement of an Event

- The probability of not event A, or P($\overline{A}$), is computed by

$$P(\overline{A}) = 1 - P(A)$$

- For example, we found the probability of getting exactly two correct answers was 3/8 or 37.5%.
- The probability of getting exactly zero, one, three, or four correct (not exactly two) is

$$P(\overline{A}) = 1 - 0.375$$
$$= 0.625 = 62.5\%$$

Number correct	Outcome
0	IIII
1	CIII ICII IICI IIIC
2	IICC ICIC ICCI CIIC CICI CCII
3	ICCC CICC CCIC CCCI
4	CCCC

exactly two correct → 2

10.1 Sample Spaces and Probability

Example: Roll two 6-sided dice

$$P(\overline{A}) = 1 - P(A)$$

Solve for the following probabilities.

a) The sum is not 6.

b) The sum is less than or equal to 9.

Possible Sums		First Die					
		⚀	⚁	⚂	⚃	⚄	⚅
Second Die	⚀	2	3	4	5	6	7
	⚁	3	4	5	6	7	8
	⚂	4	5	6	7	8	9
	⚃	5	6	7	8	9	10
	⚄	6	7	8	9	10	11
	⚅	7	8	9	10	11	12

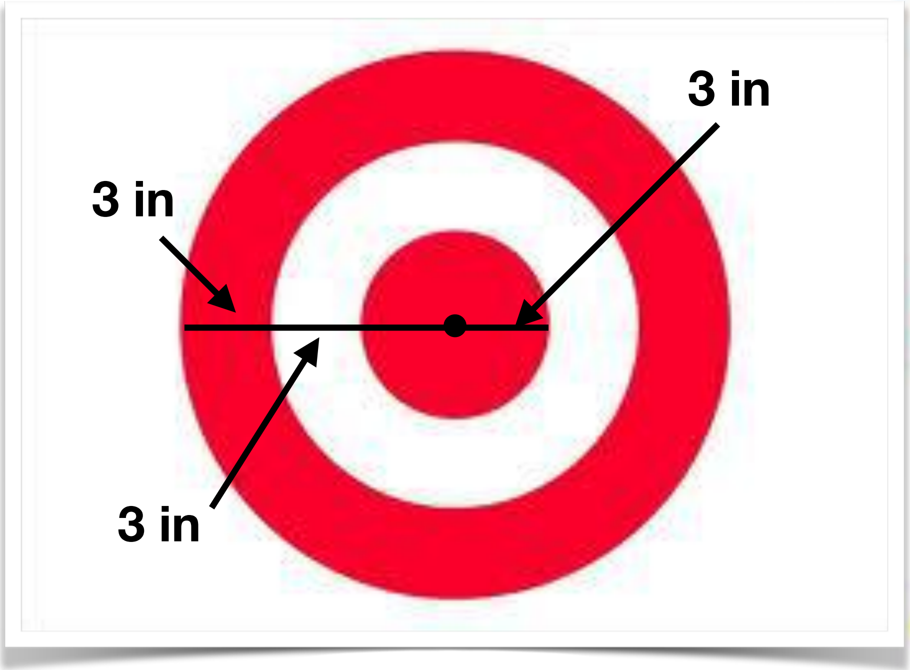
10.1 Sample Spaces and Probability

Geometric Probability

- The ratio of two lengths, areas, or volumes.

Example

- event A = hit the inside red bull's eye
- Calculate P(A)



10.1 Sample Spaces and Probability

Experimental Probability

- The results of repeated *trials* of a probability experiment.
- **Success** - A favorable outcome.

$$\text{Experimental Probability} = \frac{\text{Number of Successes}}{\text{Number of Trials}}$$

Example

- Repeated spins of the color spinner produced the following results.

Spinner Results			
red	green	blue	yellow
5	9	3	3

- Find experimental probabilities of the colors. e.g. P(red), etc.

Color Spinner

10.2 Independent and Dependent Events

Vocabulary

- **Independent Events** - Two (or more) events whose outcomes of one *<u>does not</u>* affect the other.
- **Dependent Events** - Two (or more) events whose outcomes *<u>do</u>* affect each other.

Independent or Dependent?

a) Rolling two dice.

b) Picking two numbered slips from a bag without putting any back.

10.2 Independent and Dependent Events

Probability of Independent Events

Two events A and B are independent events if and only if the probability that both events occur is the **product** of the probabilities of the events.

P(A and B) = P(A) • P(B)

Example

Rolling two 6-sided dice. What is the probability of rolling two sixes?

10.2 Independent and Dependent Events

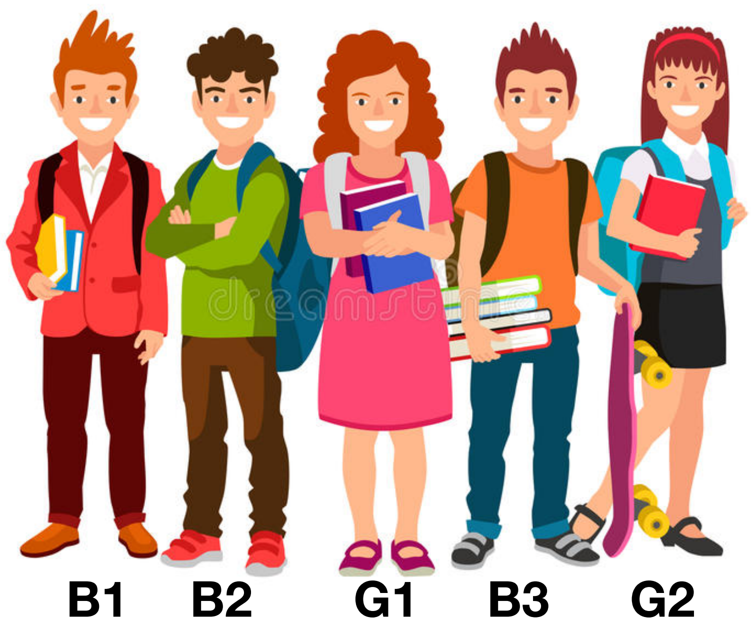
Example - Independent or Dependent?

- A group of five students include three boys and two girls. Mr Greenstein randomly selects one to be the speaker and a different student to be the recorder. Determine whether randomly selecting a boy first and randomly selecting a different boy second are independent.

Sample Set

Speaker/Recorder

B1,B2	B2,B1	B3,B1	G1,B1	G2,B1
B1,B3	B2,B3	B3,B2	G1,B2	G2,B2
B1,G1	B2,G1	B3,G1	G1,B3	G2,B3
B1,G2	B2,G2	B3,G2	G1,G2	G2,G1



10.2 Independent and Dependent Events

Example - Independent or Dependent?

- A group of five students include three boys and two girls. Mr Greenstein randomly selects one to be the speaker and a different student to be the recorder. Determine whether randomly selecting a boy first and randomly selecting a different boy second are independent.

Sample Set

Speaker/Recorder

B1,B2	B2,B1	B3,B1	G1,B1	G2,B1
B1,B3	B2,B3	B3,B2	G1,B2	G2,B2
B1,G1	B2,G1	B3,G1	G1,B3	G2,B3
B1,G2	B2,G2	B3,G2	G1,G2	G2,G1

P(first boy) = 12/20 = 3/5

P(second boy) = 12/20 = 3/5

3/5 * 3/5 = 9/25 = 36%

P(boy first and boy second) = 12/20 (1st three cols) * 3/4 (row) = 3/10 = 30%

Dependent because probabilities are not equal.

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10.2 Independent and Dependent Events

Conditional Probability

The probability that event B occurs given that event A has occurred is called the conditional probability of B given A.

P(B | A)

Example: What is the probability of choosing G1 given you already chose G2 as speaker? In other words: **P(G1 | G2)**?

Sample Set

Speaker/Recorder

B1,B2	B2,B1	B3,B1	G1,B1	G2,B1
B1,B3	B2,B3	B3,B2	G1,B2	G2,B2
B1,G1	B2,G1	B3,G1	G1,B3	G2,B3
B1,G2	B2,G2	B3,G2	G1,G2	G2,G1

B1 B2 G1 B3 G2

P(G1 | G2) = 1/4

10.2 Independent and Dependent Events

Conditional Probability

Example: A quality-control inspector checks for defective parts. The table shows the results of the inspector's work. Find (a) the probability that a defective part "passes," and (b) the probability that a non-defective part "fails."

	Pass	Fail
Defective	3	36
Non-defective	450	11

a. $P(\text{pass} \mid \text{defective}) = \dfrac{\text{Number of defective parts "passed"}}{\text{Total number of defective parts}}$

$$= \frac{3}{3 + 36} = \frac{3}{39} = \frac{1}{13} \approx 0.077, \text{ or about } 7.7\%$$

b. $P(\text{fail} \mid \text{non-defective}) = \dfrac{\text{Number of non-defective parts "failed"}}{\text{Total number of non-defective parts}}$

$$= \frac{11}{450 + 11} = \frac{11}{461} \approx 0.024, \text{ or about } 2.4\%$$

10.2 Independent and Dependent Events

Probability of Dependent Events

If two events A and B are dependent events, then the probability that both events occur is the **product** of the probability of the **first event** and the **conditional probability of the second event** given the first event.

P(A and B) = P(A) • P(B|A)

10.2 Independent and Dependent Events

Probability of Dependent Events

P(A and B) = P(A) • P(B|A)

Example

Picking two numbered slips randomly from a bag of numbered slips without putting any back.

a) What is the probability of choosing 2 and then 3?

b) What is the probability of choosing 1 or 4 and then 5?

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10.2 Independent and Dependent Events

Revisiting Conditional Probability

Start with the probability of dependent events:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Using algebra, divide each side by P(A).

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Formula for Conditional Probability

10.2 Independent and Dependent Events

Calculating Probability

Example

You randomly select 3 cards from a standard deck of 52 playing cards. What are the chances they are all hearts when:

a) you place the cards back into the deck before you choose again?
b) you do not place the cards back into the deck before choosing again?

Independent	**P(A and B) = P(A) • P(B)**
Dependent	**P(A and B) = P(A) • P(B|A)**

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